Using Multiple Power Spectrum Measurements to Sense Signals with Partial Spectral Overlap

Mihir Laghate and Prof. Danijela Cabric
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Outline

- Goals, Motivation, and Existing Work
- System Model
  - Assumptions
  - Time-Frequency Map
- Non-Negative Matrix Factorization (NNMF)
  - Challenges with existing algorithms
- Proposed Algorithm: Greedy Energy Minimizing NNMF
- MATLAB Simulation Results
- USRP Measurement Results
- Conclusions and Future Work
**Goals**

Distinguish Signals with Spectral Overlap

That is,

- Count number of signals received
- Detect sets of discrete Fourier transform bins occupied by each signal

Estimate noise power spectrum

Challenges:

- Colored noise
- Spurs and always-on interferers

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Motivating Applications

Spectral overlap by design

- IEEE 802.11b/g channels in 2.4GHz
  - Channel bonding in IEEE 802.11n

Lack of Guard Bands

- IEEE 802.11n in 5GHz bands
- LTE-Advanced


Measurements @ UCLA

# Existing Work for Distinguishing Signals

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<th>Based on</th>
<th>Blind</th>
<th>Single Antenna</th>
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<td>Transmission protocols [4],[5]</td>
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Proposed method and our prior work [1]
System Model

Wideband sensor

- Baseband bandwidth $W$ Hz
- Additive wide sense stationary Gaussian noise $\nu[t] \in \mathbb{R}_+^F$
- Welch power spectrum estimator using FFT of length $F$

Incumbent Users

- $M$ distinct frequency bands
- $m^{th}$ band has $U_m$ transmitters with freq. support $B_m$ DFT bins
  - Power spectrum received from $u^{th}$ transmitter: $X_{m,u} \in \mathbb{R}_+^F$
  - Activity $a_{m,u}[t] \in [0,1]$ is fraction of $t^{th}$ measurement that $u^{th}$ transmitter in $m^{th}$ band is active

- Received power spectrum: $Y[t] = \sum_{m=1}^{M} \sum_{u=1}^{U_m} a_{m,u}[t] X_{m,u}[t] + \nu[t]$
**Time-Frequency Map: 1 user/band**

- **Time-Freq. map** $E$ of received energy: $E = [Y[1] \ Y[2] \ldots \ Y[T]]^T$
- Define matrices: $A_{tm} = a_m[t]$, $\Sigma_{mf} = X_m(f)$, and $\Delta_{tf} = v[t](f)$

$$Y[t] = \sum_{m=1}^{M} \sum_{u=1}^{U_m} a_{m,u}[t] X_{m,u}[t] + v[t] \quad \Rightarrow \quad E = A\Sigma + \Delta$$

**Example:** $M = 3$, $F = 512$, $T = 30$

**Input: Simulated Power Spectrum**

Output computed by Non-Negative Matrix Factorization (NNMF) when given $M = 3$

**Output: Time-Freq of Each Tx**

$A(1) \times \Sigma(1)$

$+$

$A(2) \times \Sigma(2)$

$+$

$A(3) \times \Sigma(3)$
Non-Negative Matrix Factorization (NNMF)

- Let \( \hat{M} \) = Estimated number of received signals
- NNMF finds \( \hat{A} \in \mathbb{R}_+^{T \times \hat{M}}, \hat{\Sigma} \in \mathbb{R}^{\hat{M} \times F} \) to minimize \( \|E - \hat{A}\hat{\Sigma}\|^2_F \)

Challenges:

- Estimating \( \hat{M} \) is hard when \( T < F \)

- Non-convex cost function
  \[ \Rightarrow \] convergence to global minima not guaranteed

- Cost function is not probabilistic
  \[ \Rightarrow \] Not robust to noise

- Non-unique solution and \( \hat{A} \) is not binary
  \[ \Rightarrow \] \( \hat{\Sigma} \neq \Sigma \), i.e., thresholding \( \hat{\Sigma} \) will not detect all occupied DFT bins
Prior Work: NNMF-based Algorithm

Initialization $\hat{M} = 1$

Energy Detection

$E' = \text{NNMF of } E'$ with $\hat{M}$ signals

Increment $\hat{M}$

No

Noise band detected?

Yes

Detect Occupied Bands

$\hat{B}_1, \hat{B}_2, ..., \hat{B}_M \subseteq \{0, ..., F - 1\}$

Reconstructed Factors for $\hat{M} = 4$

“Leaked” energy

Significant signal energy

Noise Band
Motivating NNMF Algorithm: Robust XRay

\[
E[Y[t] | a[t]] = \sum_{m=1}^{M} \sum_{u=1}^{U_m} a_{m,u}[t] E[X_{m,u}[t]] + E[v[t]]
\]

i.e., cone with received power spectra as extreme rays

Recursive Algorithm:
1. Choose point with maximum residual as extreme ray
2. Measure residual to all measurements
3. Repeat 1 until all measurements lie within cone

Separability Assumption

\[ E[Y(t) | a(t)] = \sum_{m=1}^{M} \sum_{u=1}^{U_m} a_{m,u}(t) E[X_{m,u}(t)] + E[v(t)] \]

i.e., cone with received power spectra as extreme rays

Separability Assumption:
Requires at least one measurement of each extreme ray

- At least one noise only measurement
- For each transmitter, at least one measurement where it is the only active transmitter

Proposed Greedy Energy Minimizing NNMF

Noise power spectrum learned from data:
1. Initialize noise estimate as min. energy measurement.
2. Estimate noise as mean of measurements with Mahalanobis distance \(< \tau\) to est. noise distribution.
3. Repeat Step 2 until convergence.

Received Signal

Welch Power Spectrum Estimator

Append to Time Frequency Map \(E\)

Detect Noise-Only Measurements \(\mathcal{L}\)

Index in \(\{1, \ldots, T\} \setminus \mathcal{L}\) with Lowest Energy labelled as Detected Signal

\(\mathcal{L} \triangleq \) Measurements that are linear combinations of Detected Signals

\(|\mathcal{L}| = T?\)

Yes

Energy Detection on Detected Signals

\(\triangleright\) Center Frequency
\(\triangleright\) Bandwidth

No
Proposed Greedy Energy Minimizing NNMF

1. Initialize noise estimate as min. energy measurement
2. Estimate noise as mean of measurements with Mahalanobis distance < τ to est. noise distribution
3. Repeat Step 2 until convergence

Constrained least squares minimization estimates activity
1. Activity for noise ∈ (−∞, 1]
2. Activity for detected signals ∈ [0, ∞)

- Measure power spectrum
- Detect noise-only measurements
- Append to time frequency map
- Detect noise-only measurements
- Index in {1,...,T}\mathcal{L} with lowest energy labelled as detected signal

\mathcal{L} ≜ \text{Measurements that are linear combinations of detected signals}

|\mathcal{L}| = T?

Energy detection on detected signals

Center frequency
Bandwidth
Performance Metrics

- Number of detected bands
- Number of extra bands detected
- Relative Errors in Center Frequency and Bandwidth
Performance Metrics

- Number of detected bands
- Number of extra bands detected
- Relative Errors in Center Frequency and Bandwidth

Edge Weights: \[ \delta\left(B_{m_1}, \hat{B}_{m_2}\right) = F - \left|B_{m_1} \oplus \hat{B}_{m_2}\right| \]

Symmetric Difference

Our Output

Ground Truth

Computed using Maximum Weight Matching

Fully Connected Bipartite Graph
MATLAB Simulations: Performance vs. $T$

Receiver:
- Bandwidth 100 MHz
- 512 length FFT, average of 64 windowed overlapping segments
- Up to 50 measurements, i.e., 8.32ms
- Parameters: $P_{fa} = 0.01$, $\tau_r = 0.1$

802.11g Transmitters:
- Each network = 1 AP + 2 STAs
- Channels 1, 4, 6, 8, 11
- Saturated uplink and downlink flows
- Shadow fading channels with 6dB variance

**Number of Detected Signals**

**Number of Extra Signals**

- #Signals
- GEM-NNMF
- NNMF-based
- Robust XRay

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Simulation: Performance vs. Spectral Overlap

Number of Detected Signals

Overlap Ratio

Number of Extra Signals

Frequency Error

Relative Error in Center Frequency

Relative Error in Bandwidth

Overlap Ratio

#Signals 2 3 4 5
GEM-NNMF
NNMF-based
Robust XRay
USRP Measurements: Example

- Device: USRP N210 with CBX daughtercard
- Measurements at 2.452 GHz
- 8-bit samples @ 50 MS/s

**802.11 Signals Detected**

Black arrows: detected supports
Labels: corresponding 802.11 channels
Multiple USRP Measurements

- Contour plot of 2D histogram of detected center frequencies and bandwidths
- 1000 realizations of 50 MHz measurements @ 2.452 GHz at UCLA

Android WiFiAnalyzer used to confirm 802.11 channels 6, 7, 8, and 11 in use
Conclusions and Future Work

- Noise power spectrum can be estimated automatically when sensing communicating incumbent users.

- Multiple power spectrum measurements can distinguish real world spectrally overlapped signals even in unknown channels.

- Conventional signal detection and estimation theory may not be sufficient.

Future Work:

- Find structural properties of optimization problem to reduce computational complexity.

- Estimate time of activity, i.e., $\hat{A}$, for use in traffic estimation.
Thank you!

Questions?

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Selected References


