## THE IMPACT OF SMALL-CELL BANDWIDTH REQUIREMENTS ON STRATEGIC OPERATORS

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## 5G Trends

$\square$ Heterogeneous networks

- Cells (Macro/Small)



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## Spectrum Sharing


$\square 100 \mathrm{MHz}$
$\square$ Shared with naval radar
$\square$ Three-tier sharing rules

- Incumbents
- Priority Access Licenses
- General Access
$\square$ Low power
$\rightarrow$ small cells


## Spectrum Sharing


$\square 100 \mathrm{MHz}$
$\square$ Shared with naval radar
$\square$ Three-tier sharing rules
$\square$ Low power
$\rightarrow$ small cells

How will the low power / small-cell requirement affect prices, bandwidth allocation, and social welfare?

## Assumptions

$\square$ SPs manage two networks:

- Macro-cell / Small-cell
$\square$ Two types of users: mobile / fixed
- Mobile users must connect to macro-cell network
- Fixed users can connect to macro- or small-cell network
$\square$ Utility is a function of the rate received
- Shared spectrum
$\rightarrow$ bandwidth (rate) is split evenly among users


## Assumptions

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- Mobile users must connect to macro-cell network
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$\square$ Utility is a function of the rate received
$\square$ Each SP must provide a minimum amount of bandwidth for small cells.


## Related Work

$\square$ Chen et al:

- Workshop on Smart Data Pricing, 2015

Model for competing service providers

- Infocom, 2016

Licensed and unlicensed spectrum
$\square$ Differences from other related work:
$\square$ Two classes of users (mobile/fixed)
$\square$ Providers set prices and optimize bandwidth

- Constraint on minimum small-cell bandwidth


## Model



Supply
Demand

## Model



Supply
Demand

## Model



How do the small cell constraints affect bandwidth and prices?

## Main Results (1)

An equilibrium always exists and is unique.
Adding the constraints can only decrease social welfare ( $\alpha$-fair utilities).

## Adding Small-Cell Bandwidth


$\square$ SPs have exclusive-use bands $B_{1}$ and $B_{2}$, which can be split between macro and small cells.
$\square$ Add bandwidth B designated for small cells.

## Social Welfare: Large $B$



Dyspan 2017, Baltimore, MD

## Social Welfare: Smaller B



Dyspan 2017, Baltimore, MD

## Main Results (2)

$\square$ An equilibrium always exists and is unique.
$\square$ Possible effect of adding constraint on equilibrium:


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## Effect of Constraint on Equilibrium



## Utility

$\square$ Utility for each user is a function of the rate $r$.
$\square$ Total rate (capacity) depends on spectral efficiency $R_{0}$

- Macro-cell capacity for SP i: $C_{i, M}=B_{i, M} R_{0}$
$\square$ Small-cell capacity for $S P$ i: $C_{i, S}=\lambda_{s} B_{i, s} R_{0}$
$\lambda_{s}>1$ accounts for higher density and/or spectral efficiency of small-cell network


## Utility

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- Macro-cell capacity for SP i: $C_{i, M}=B_{i, M} R_{0}$
$\square$ Small-cell capacity for SP i: $C_{i, S}=\lambda{ }_{s} B_{i, s} R_{0}$
$\square$ Will assume the class of $\alpha$-fair utility functions:

$$
u(r)=\frac{r^{1-\alpha}}{1-\alpha} \quad \begin{aligned}
& \alpha \rightarrow 0, u(r) \text { becomes linear } \\
& \alpha \rightarrow 1, u(r) \text { becomes logarithmic }
\end{aligned}
$$

## Sequential (Two-Stage) Game

1. SPs set bandwidths $\quad B_{i, M} \quad B_{i, S}$
2. SPs set prices $\quad p_{i, M} \quad p_{i, S}$

Fixed users choose network to maximize surplus (utility minus cost): $S(r)=u(r)-p r$ rate $r^{*}=\arg \max S(r)=D(p) \quad$ (demand function)

We will characterize sub-game perfect Nash equilibria:

1. Price equilibrium / user association given bandwidth allocation.
2. Bandwidth allocation given that prices are set according to 1 .

## Revenue Maximization

$$
\max S_{i}=K_{i, M} p_{i, M} D\left(p_{i, M}\right)+K_{i, S} p_{i, S} D\left(p_{i, S}\right)
$$

$$
\text { subject to } K_{i, M} D\left(p_{i, M}\right) \leq C_{i, M}
$$

$$
K_{i, S} D\left(p_{i, S}\right) \leq C_{i, S}
$$

$$
B_{i, M}+B_{i, S} \leq B_{i}
$$

in macro-/small-cell networks

$$
\begin{aligned}
& 0 \leq p_{i, M}, p_{i, S}<\infty \\
& B_{i, M} \geq 0, \quad B_{i, S} \geq B_{i, S}^{0}
\end{aligned}
$$

## Social Welfare (Utility) Objective

$$
S W=\sum_{i=1}^{N} K_{i, M} u\left(r_{i, M}\right)+K_{i, S} u\left(r_{i, S}\right)
$$

With $\alpha$-fair utility functions the equilibrium maximizes SW without small-cell bandwidth constraints.

## Social Welfare Loss

$\square$ SW loss occurs when

$$
\frac{N_{f} \lambda_{S}^{1 / \alpha-1}}{N_{f} \lambda_{S}^{1 / \alpha-1}+N_{m}} \sum_{i \in \mathcal{N}} B_{i}<\sum_{i \in \mathcal{N}} B_{i, S}^{0}
$$

$\square$ The loss satisfes:

$$
\frac{\mathrm{SW}_{\mathrm{w}}^{\mathrm{NE}}}{\mathrm{SW}_{\mathrm{wo}}^{*}} \geq\left(\frac{N_{f} \lambda_{S}^{1 / \alpha-1}}{N_{m}+N_{f} \lambda_{S}^{1 / \alpha-1}}\right)^{\alpha}
$$

$\square$ Equality holds when $B_{i, S}{ }^{0}=B_{i}$ for every SP i.

## Constraining New Bandwidth

$\square$ Given new bandwidth $B$, there a exists a threshold $T$ such that if $B>T$, constraining $B$ for small cells reduces SW.

$$
T=\frac{\left(B_{1}^{o}+B_{2}^{o}\right) N_{f} \lambda_{S}^{1 / \alpha-1}}{N_{m}}
$$

$\square$ If $B<T, B$ can be split between SPs 1 and 2 so that the competitive equilibrium achieves the maximum SW.

## Social Welfare: Smaller B



## Conclusions

$\square$ Adding constraints on small-cell bandwidth can change competitive equilibrium and lead to a loss in SW.
$\square$ The constraint may cause an SP to reduce its smallcell bandwidth, although the total allocation cannot decrease.
$\square$ Constraining new bandwidth $B$ leads to inefficient allocations when $B$ exceeds a threshold.

