Coverage and Rate Analysis of Super Wi-Fi Networks Using Stochastic Geometry

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Questions to ponder over ..

- How can TV White Spaces (TVWS) be used to provide extensive broadband connectivity to rural/under-served areas?

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- How do regulations imposed by FCC affect network performance (particularly transmit power and deployment height restrictions)?
- Given a region of interest, what should be the -
  1. deployment density,
  2. transmit power, and
  3. deployment height

  of base stations to achieve 'reasonable' performance?

Problem Statement

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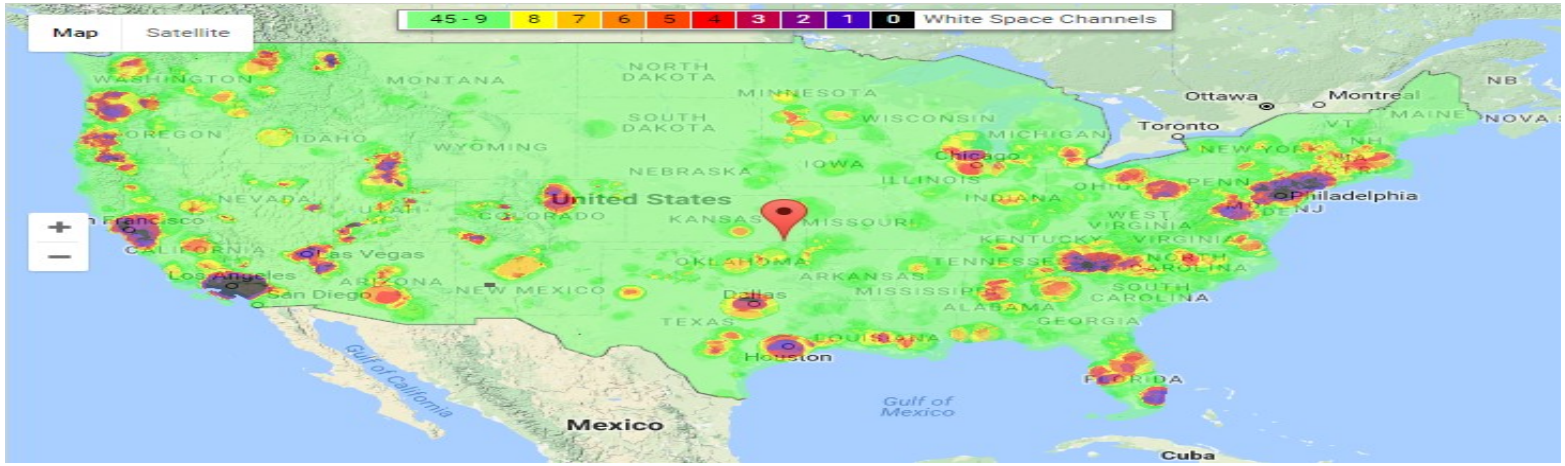
- What is Super Wi-Fi? - Wi-Fi-like networks operating in TVWS bands
- What is super about Super Wi-Fi? - TVWS frequencies have excellent propagation characteristics including low pathloss, and wall penetration losses

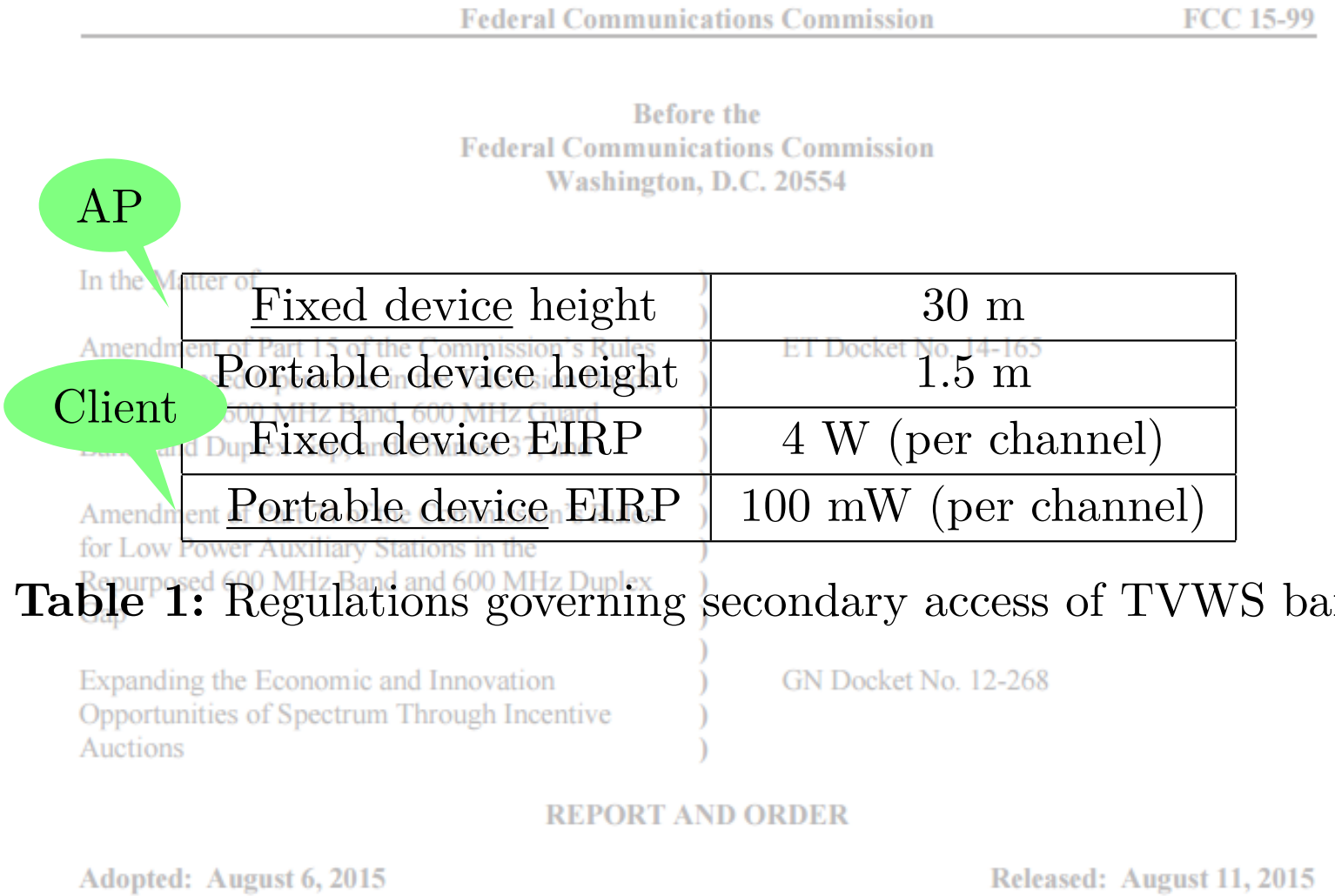
TVWS Availability



TV Channels	Freq. Band	Freq (MHz)	Allowed Devices
2	VHF	54-60	Fixed
5-6	VHF	76-88	Fixed
7-13	VHF	174-216	Fixed
14-20	UHF	470-512	Fixed
21-35	UHF	512-602	Fixed and Portable
39-51	UHF	620-698	Fixed and Portable

[0]From Google spectrum database (as of 02/16/2017)

FCC Regulations


Federal Communications Commission FCC 15-99

Before the
Federal Communications Commission
Washington, D.C. 20554

In the Matter of)
Amendment of Part 15 of the Commission's Rules) ET Docket No. 14-165
for Low Power Auxiliary Stations in the)
Repurposed 600 MHz Band and 600 MHz Duplex)
Gap)
)
Expanding the Economic and Innovation) GN Docket No. 12-268
Opportunities of Spectrum Through Incentive)
Auctions)

REPORT AND ORDER

Adopted: August 6, 2015 Released: August 11, 2015

AP

Client

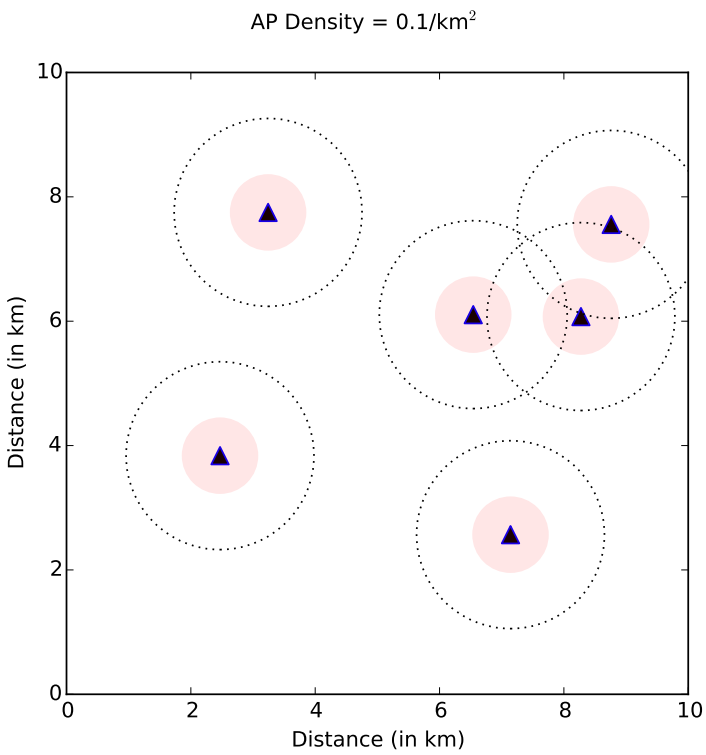
Fixed device height	30 m
Portable device height	1.5 m
Fixed device EIRP	4 W (per channel)
Portable device EIRP	100 mW (per channel)

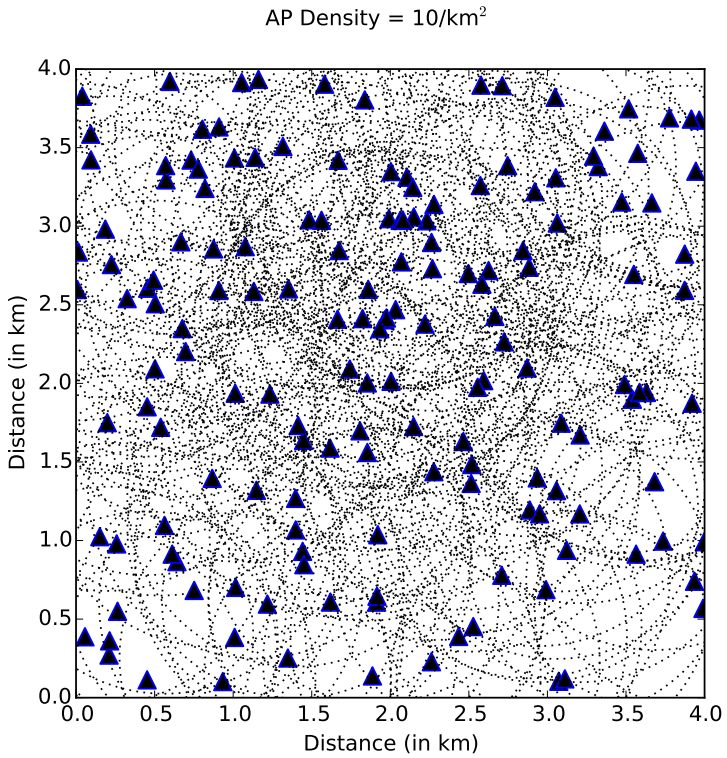
Table 1: Regulations governing secondary access of TVWS bands

Transmit Power Asymmetry

$$P_{AP} = 4\,W, h_{AP} = 30\,m, P_C = 100\,mW, h_C = 1\,m$$

Black dotted circles - downlink coverage, Red circles - uplink coverage





Downlink coverage - Distance till AP transmission power > Threshold_{DL}
Uplink coverage - Distance till client transmission power > Threshold_{UL}

Objectives

Analyze the coverage and rate performance of a Super Wi-Fi network as a function of the following quantities:

1. density of AP distribution (λ)
2. AP transmit power (P_{AP})
3. AP height (h_{AP})
4. power asymmetry (uplink viability)

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Tool - Stochastic Geometry

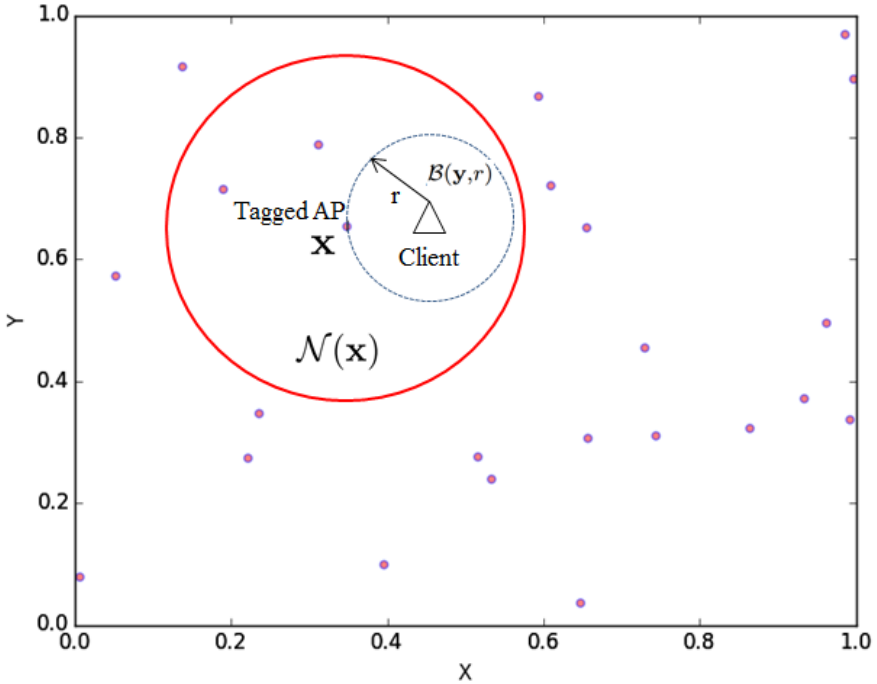
Methodology

Model AP distribution as a homogeneous Poisson point process (PPP) and analytically characterize coverage and area spectral efficiency (ASE) of the network

Notations and Definitions

- Φ_A - Set of AP locations - Homogeneous PPP
- λ - Intensity of PPP (Density of AP deployment) in per km^2
- σ - CCA Threshold
- $\rho(\mathbf{x}, \mathbf{y})$ - Pathloss between points $\mathbf{x}$ and $\mathbf{y}$
- $G(\mathbf{x}, \mathbf{y})$ - Fading between points $\mathbf{x}$ and $\mathbf{y}$
- $\mathrm{P_{AP}}$ - AP transmit power
- $\mathrm{P_C}$ - Client transmit power

Modeling of CSMA



- Each point $\mathbf{x}$ in Φ_A will be assigned an independent mark $t_{\mathbf{x}}$ uniformly distributed in [0,1] representing the *back-off time*
- The subset of concurrently transmitting APs - $\Phi_T = \{\mathbf{x} \in \Phi_A : t_{\mathbf{x}} < t_{\mathbf{y}}, \forall\, \mathbf{y} \in \mathcal{N}(\mathbf{x})\}$

Now into Math

The throughput of an AP averaged out over all AP - *uplink viable* client distances r is given by -

$$\mathcal{T} = \mathbb{E}_r(\mathcal{T}(r)) = \int_0^\infty \underbrace{\mathcal{T}(r)}_{\text{Throughput of AP at r}} \times \underbrace{f_R(r|I_u = 1)}_{\text{AP-Client distance distribution}} dr$$

- $f_R(r|I_u = 1) = \frac{f_R(r)\mathbb{P}(I_u=1|R=r)}{\mathbb{P}(I_u=1)}$
- $f_R(r) = 2\pi r \lambda e^{-\lambda \pi r^2}$
- $I_u = \mathbb{1}(\underbrace{P_C\, \rho(r)\, G(r)}_{\substack{\text{Received power at AP} \\ \text{from client}}} > \gamma)$

$$\implies \mathbb{P}(I_u = 1|R = r) = \mathbb{P}\left(G(r) > \frac{\gamma}{\mathrm{P_C}\, \rho(r)}\right) = e^{\frac{-\mu\gamma}{\mathrm{P_C}\, \rho(r)}}$$

Math continued ..

$\mathcal{T}(r)$ - throughput of the tagged AP conditioned on r = Product of :

1. transmission probability of AP conditioned on r : $p_T(r)$
2. expected rate at the client associated with the tagged AP : $(\mathbb{E}[log_2(1 + \text{SINR})])$

$$\therefore \mathcal{T} = \int_0^\infty \mathcal{T}(r) f_R(r|I_u = 1)\, dr$$
$$= \int_0^\infty p_T(r)\mathbb{E}[log_2(1 + \text{SINR})] f_R(r|I_u = 1)\, dr$$

Numerical Results and Discussion

Key Parameters and Range of Values

Channel Center Frequency	600 MHz
Channel Bandwidth	6 MHz
Number of channels	1
AP distribution	Homogeneous PPP of intensity λ
AP Transmission Power	Variable between 0.1 W and 4 W
Client Transmission Power	0.1 W
AP Height	Variable between 1.5 m and 30 m
Client Height	1 m
Pathloss	Based on ITU-R P.1411-8
Fading	Exponentially distributed with mean 1
CCA Threshold σ	-82 dBm
Uplink Viability Threshold γ	-82 dBm
Noise Variance (N_0)	-173.97 dBm/Hz
Traffic model	Persistent downlink

Pathloss Modeling (ITU-R P.1411-8)

Model is dual-slope and is a function of transmitter/receiver antenna heights

$$\rho(\mathbf{x},\mathbf{y}) \text{ in dB} = \begin{cases} \rho_{\text{LOS}} + 20 + 25\log\left(\frac{d}{R_{bp}}\right), \text{ if } d < R_{bp} \\ \rho_{\text{LOS}} + 20 + 40\log\left(\frac{d}{R_{bp}}\right), \text{ if } d \geq R_{bp} \end{cases}$$

- ρ_{LOS} is the line-of-sight pathloss (in dB), given by

$$\rho_{\text{LOS}} = \left| 20\log\left(\frac{\lambda^2}{8\pi h_t h_r}\right) \right|$$

- R_{bp} is the breakpoint distance (in meters), given by

$$R_{bp} = \frac{1}{\lambda}\sqrt{(4h_t h_r)^2 - 4(h_t^2 + h_r^2)\left(\frac{\lambda}{2}\right)^2 + \left(\frac{\lambda}{2}\right)^4}$$

Two Pathloss Models

To obtain R_{bp} and ρ_{LOS}:

- AP - AP interaction - Use $h_t = h_r = h_{\text{AP}}$
- AP - Client interaction - Use $h_t = h_{\text{AP}}$ and $h_r = h_{\text{C}}$

Why Stochastic Geometry?

- Analytical characterizations using stochastic geometry give flexibility in changing operating parameters without re-running long simulations
- Our model is validated with OPNET (industry-standard packet-level simulator)

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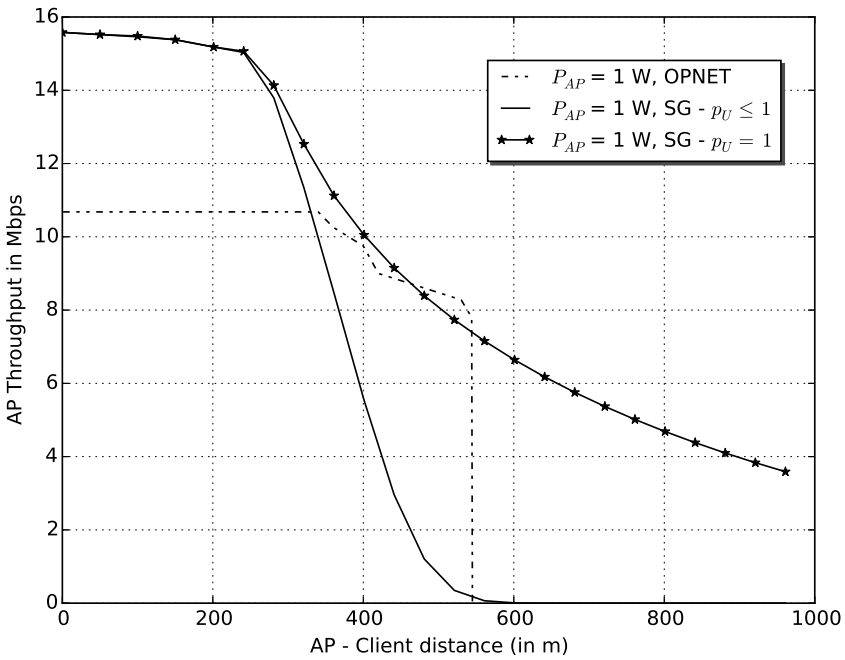
Results from stochastic geometry-based models and OPNET simulation. The plot with $p_U \leq 1$ is for the case when uplink viability is probabilistic and the one with $p_U = 1$ is when uplink is assumed to be always viable



Probability of AP Transmission

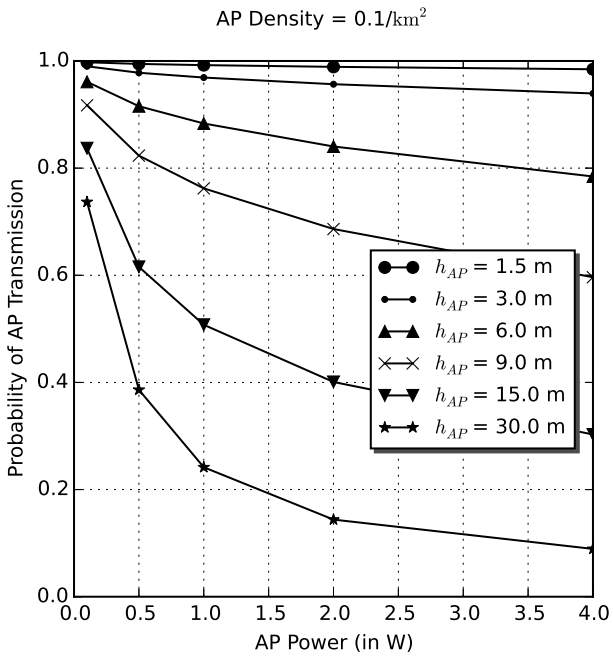


Figure 1: Low deployment density

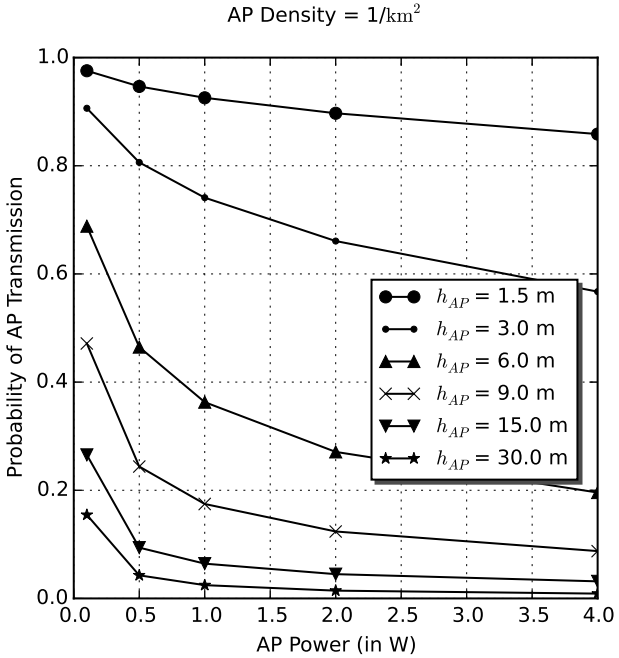


Figure 2: Medium deployment density

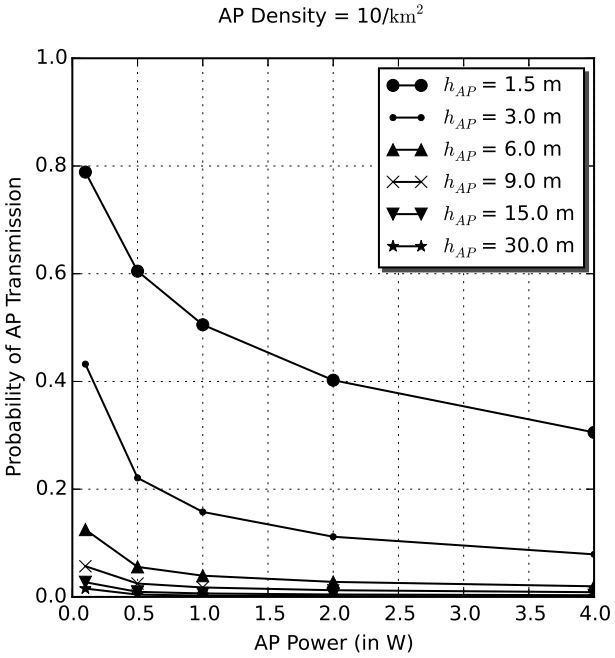


Figure 3: High deployment density

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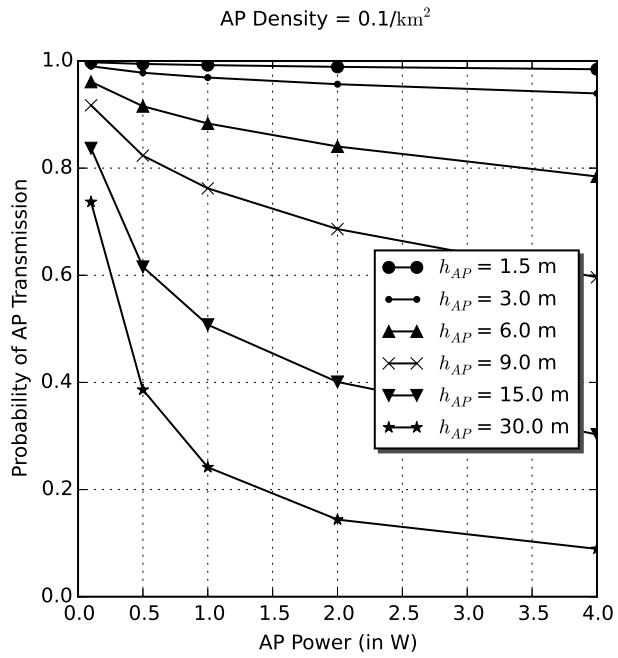

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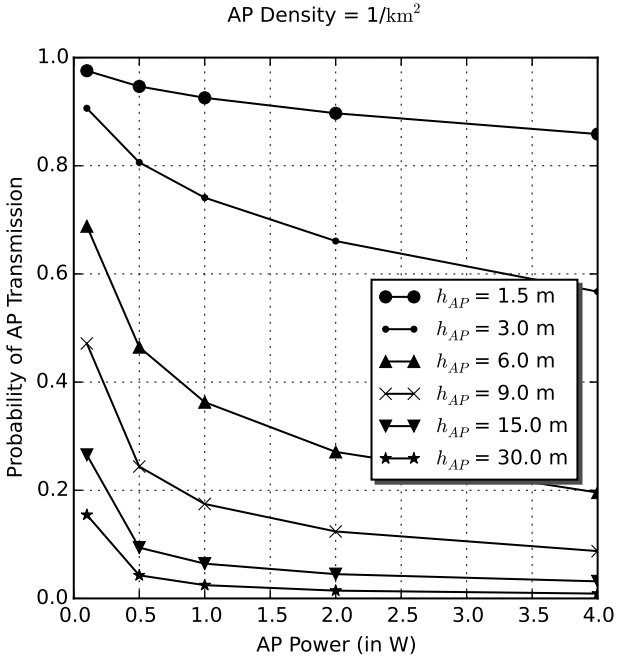

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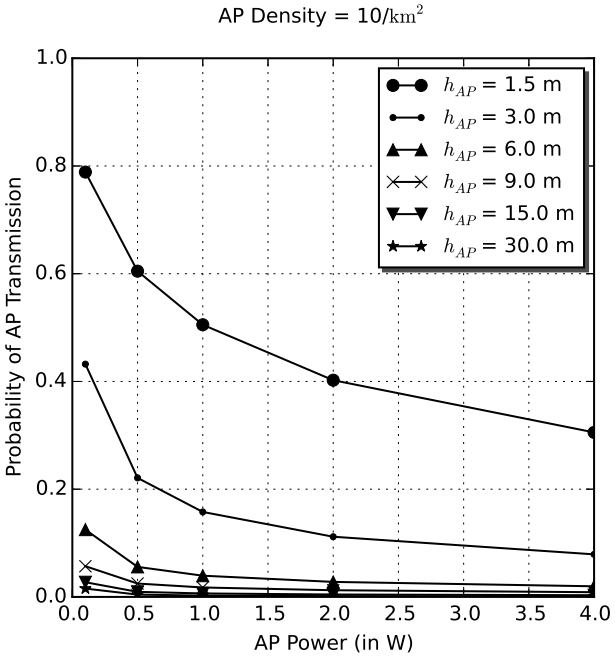

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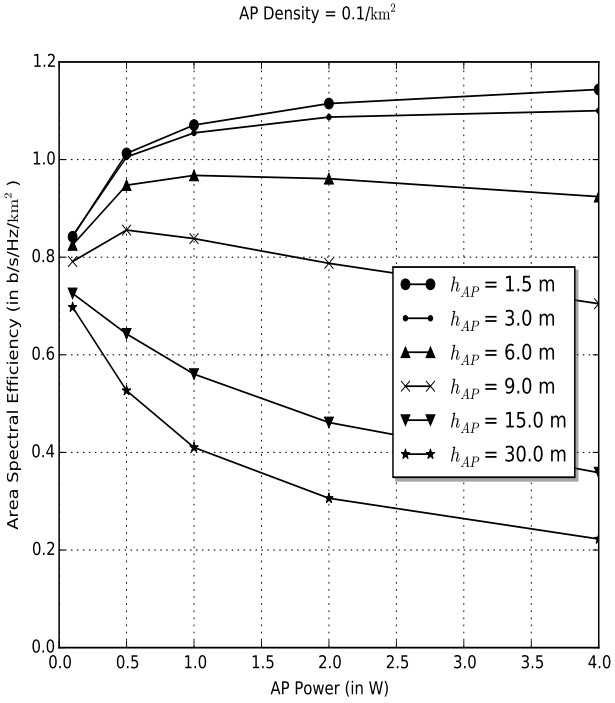
Takeaway

Stay Low! Higher AP heights are detrimental!

Downlink Area Spectral Efficiency



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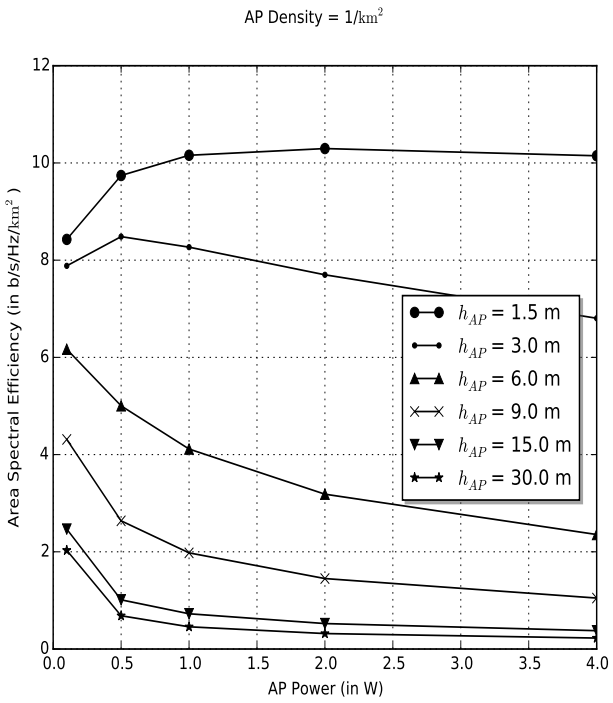
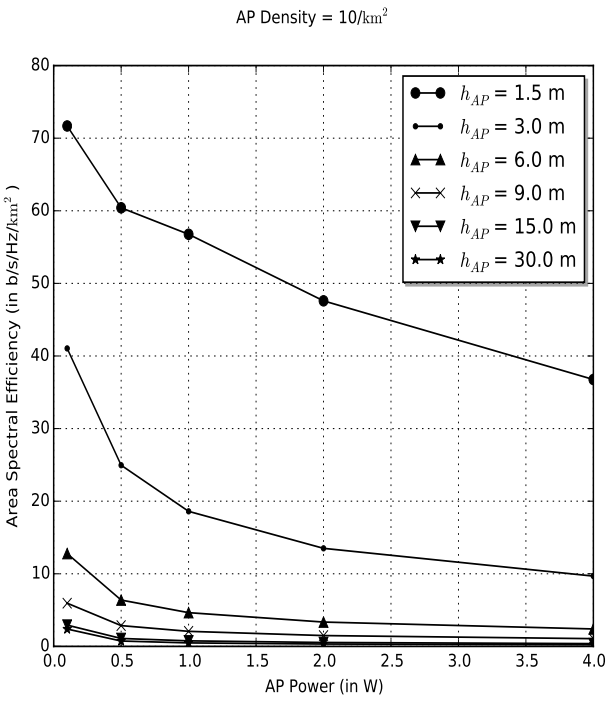


Figure 5: Medium deployment density



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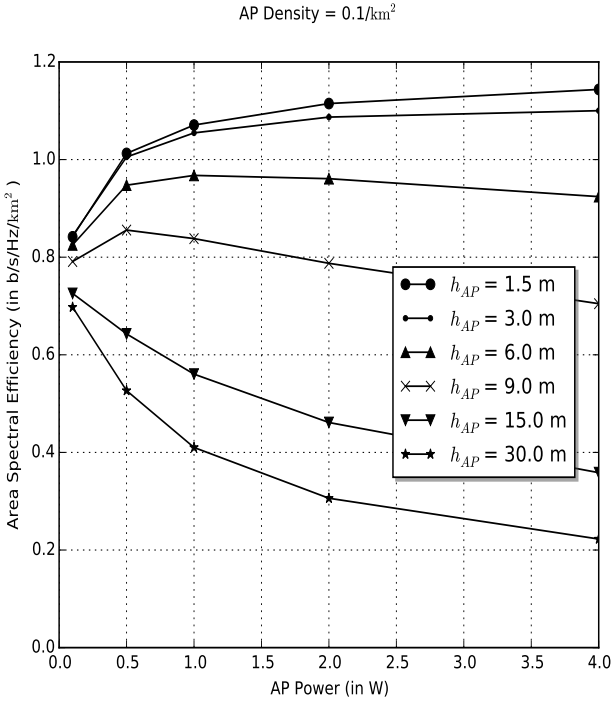

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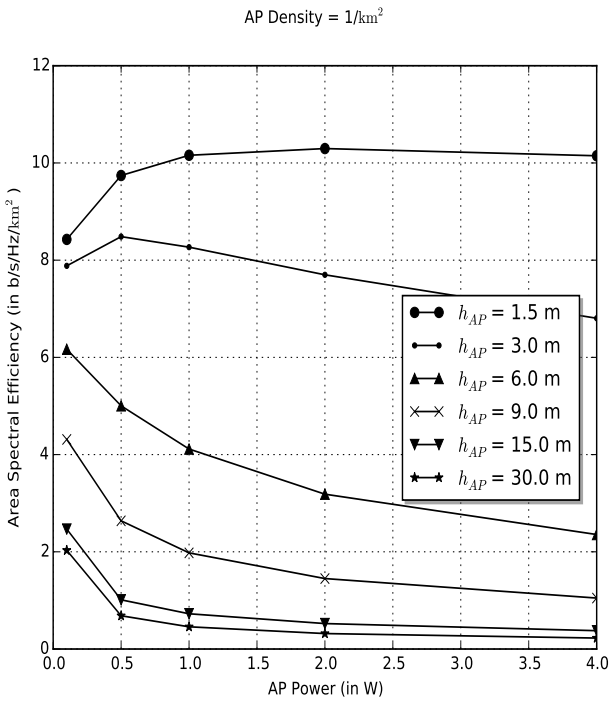

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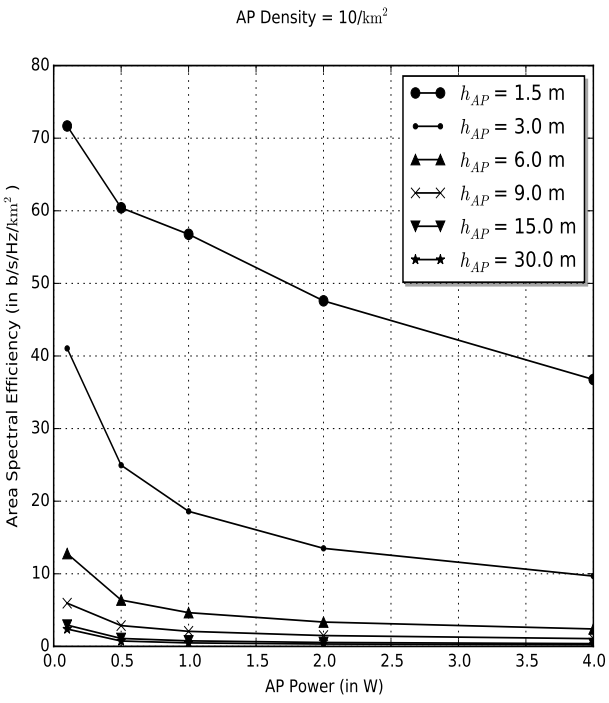

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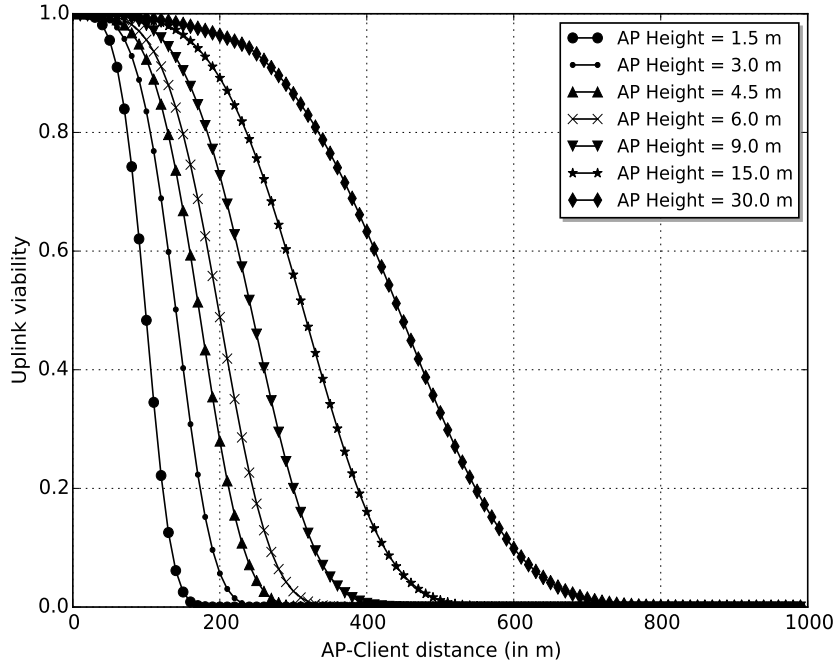

Figure 7: Probability of uplink viability vs AP-client distance r for various AP heights

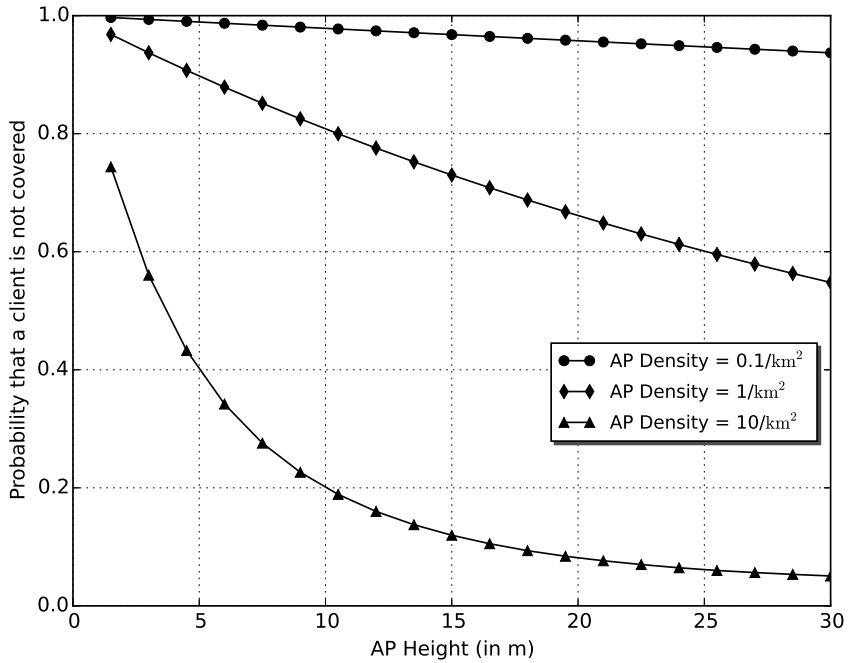

Figure 8: Probability that client does not find an AP to associate with; note that $P_C = 0.1$ W always

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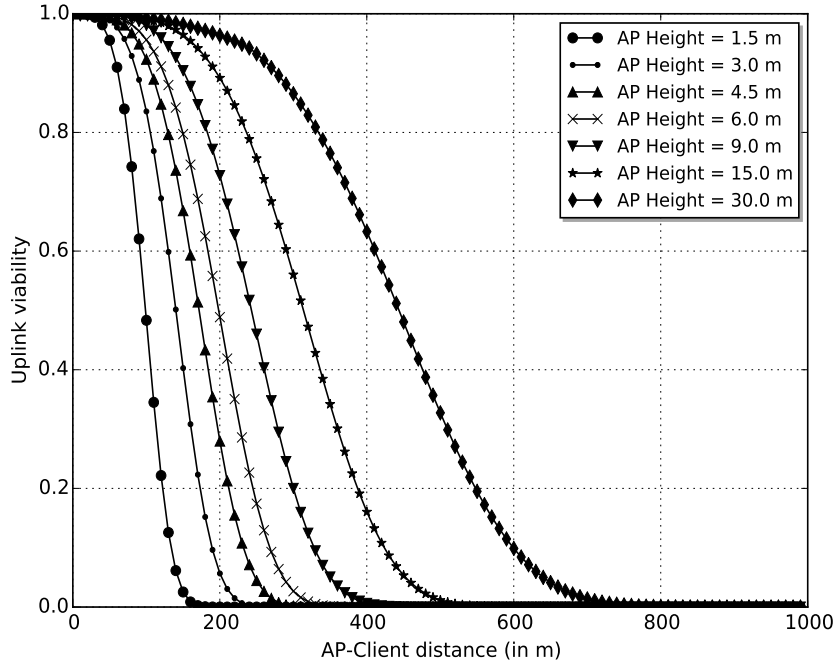

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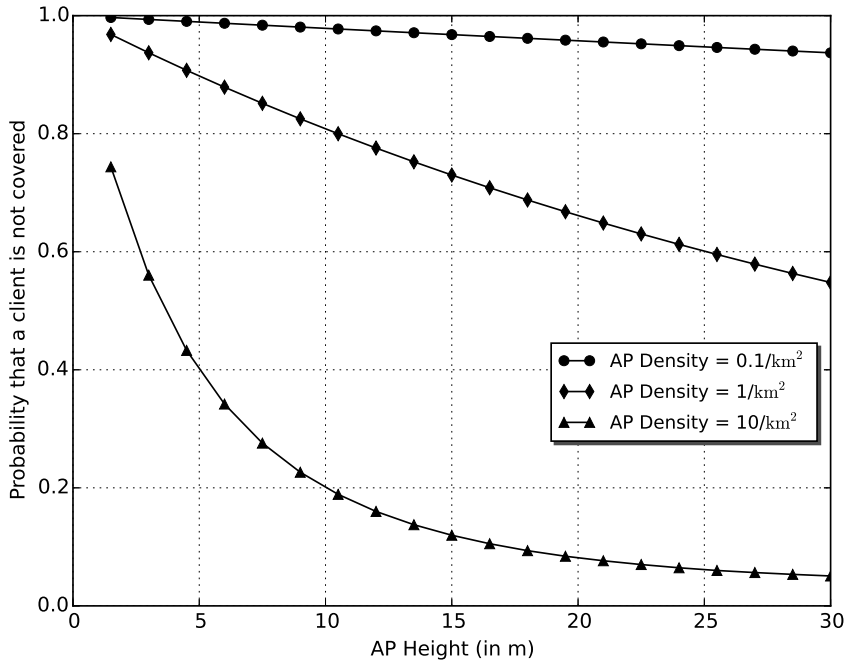

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Takeaway

Go High! Higher AP heights lead to pervasive coverage!

Prescription for AP Operating Parameters

Priority	P_{AP}	h_{AP}
Rate	1 - 4 W	1.5 - 3 m
Coverage	0.1 W	30 m

Low AP density ($0.1/km^2$)

Priority	P_{AP}	h_{AP}
Rate	1 - 4 W	1.5 m
Coverage	0.1 W	30 m

Medium AP density ($1/km^2$)

Priority	P_{AP}	h_{AP}
Rate	0.1 W	1.5 m
Coverage	0.1 W	10 - 30 m

High AP density ($10/km^2$)

Sharon Springs, Kansas

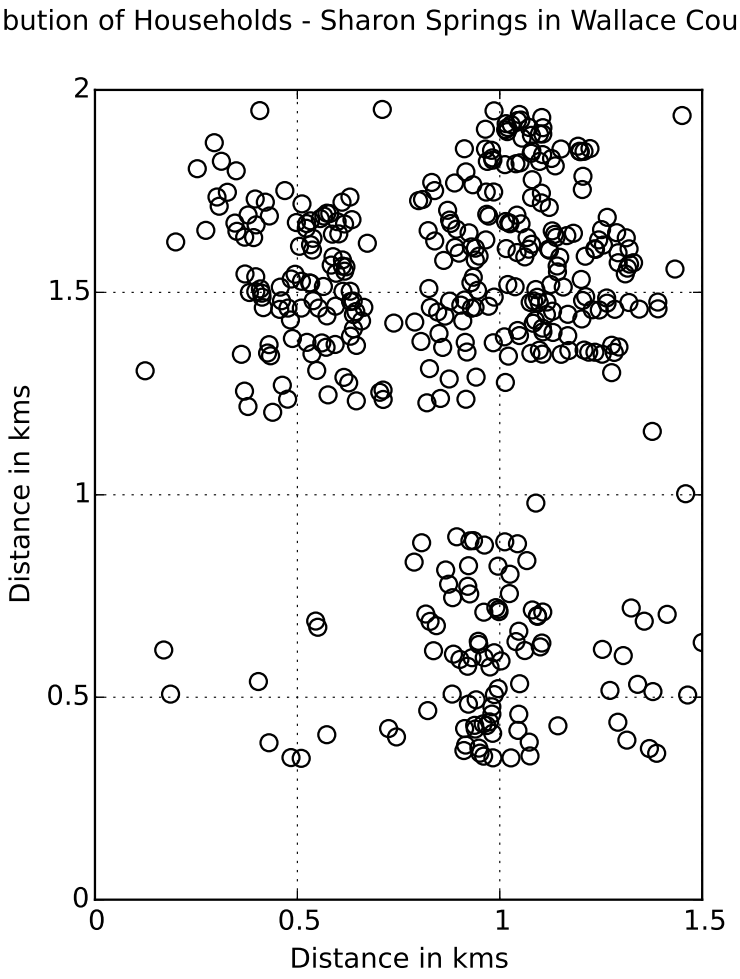
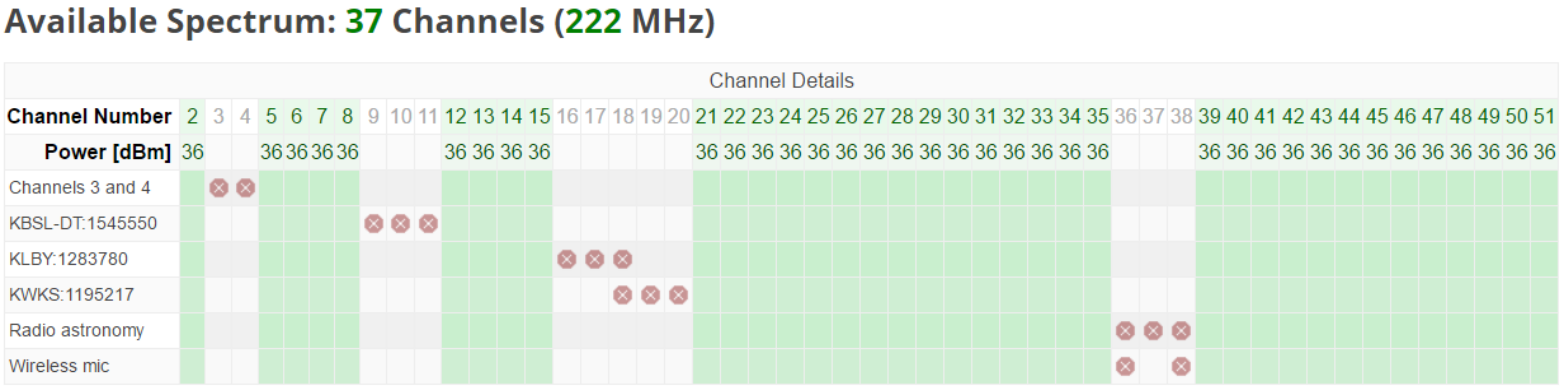


Figure 9: Obtained from 2010 US Census (Each circle represents a household)

Sharon Springs - Network Specifications

- Data Rate Requirements
  - Number of households - ~ 400 in an area of 3 km^2
  - Data requirement of each house - 10 Mbps
    $\implies$ Total network throughput $= 1330\,\text{Mbps/km}^2$
- Channel Availability



Availability of TVWS channels in Sharon Springs from Google spectrum database - $h_{AP} > 6$m

Sharon Springs - Network Design

- Deployment density = 10 APs/km^2
- Assuming coverage probability of 75% suffices, set $h_{AP} = 6$m (from fig. 12)
- For this choice of h_{AP}, choose P_{AP} which yields the highest ASE from fig. 8-10
  In this case, $P_{AP} = 0.1$ W with ASE = 12 b/s/Hz/km^2
- Channel throughput = 72 Mbps/km^2 $\implies$ Use 18 of the available 37 channels

Concluding Remarks

- FCC regulations on height and transmit power of fixed/portable devices lead to asymmetry in downlink and uplink coverage

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- Albeit permitted by FCC, operating APs at high transmit power and height is not meritorious even at densities as low as $1/10\,\mathrm{km}^2$
- Clients have better uplink viability when APs are deployed at higher heights
- Choice of operating parameters depends on desired balance between rate and coverage

BACKUP SLIDES

Probability of Transmission of an AP

$$\mathcal{T} = \int_0^\infty p_T(r)\mathbb{E}[log_2(1+\text{SINR})]f_R(r|I_u=1)\,dr$$

Probability of Transmission of the Tagged AP

$$p_T(r) = \int_0^1 e^{-\lambda_a t_0 \int_{\mathbb{R}^2 \setminus \mathcal{B}(\mathbf{y},r)} S(\mathbf{x})\, d\mathbf{x}} dt_0 = \frac{1 - e^{-\lambda_a \int_{\mathbb{R}^2 \setminus \mathcal{B}(\mathbf{y},r)} S(\mathbf{x})\, d\mathbf{x}}}{\lambda_a \int_{\mathbb{R}^2 \setminus \mathcal{B}(\mathbf{y},r)} S(\mathbf{x})\, d\mathbf{x}}$$

$S(\mathbf{x})$ is the probability of AP at $\mathbf{0}$ to sense another AP located at $\mathbf{x}$:

$$S(\mathbf{x}) = \mathbb{P}[\underbrace{P(\mathbf{0},\mathbf{x})}_{\text{Power received at } \mathbf{0} \text{ from } \mathbf{x}} > \sigma] = \mathbb{P}[\mathrm{P}_{\mathrm{AP}} \cdot \rho(\mathbf{0},\mathbf{x}) \cdot G(\mathbf{0},\mathbf{x}) > \sigma]$$

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Intuitive Reasoning

The intensity of PPP of APs that can potentially prevent AP at $\mathbf{0}$ from transmitting is $\lambda_a t_0$ ($mark < t_0$). This PPP is further thinned by $S(\mathbf{x})$ which is location-dependent.

SINR Distribution and Expected Rate Computation

$$\mathcal{T} = \int_0^\infty p_T(r)\mathbb{E}[log_2(1+\text{SINR})]f_R(r|I_u=1)\,dr$$

SINR Distribution - Preliminaries

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- 'Signal' part in SINR = $\mathrm{P_{AP}}\rho(r)G$ (where the AP-Client distance is r) = Straight forward result
- 'Noise' part in SINR = kTB $\approx -106\,\mathrm{dBm}$ at B = 6 MHz
- 'Interference' part in SINR - Not a trivial computation as this quantity depends on:
  1. Probability of Transmission of AP
  2. Distribution of interfering APs around the tagged APs
     - which implicitly depends on the CCA threshold σ
  3. Fading coefficient
  4. Pathloss

Distribution of SINR

Interfering APs form an *in-homogeneous* PPP, the local intensity of which depends on distance x from the tagged AP
Laplace Transform of SINR,

$$\psi_{I+N}(s) = \psi_I(s)\psi_N(s) \approx e^{-\lambda_a \int_0^{2\pi} \int_r^{\infty} h(b(\rho,\theta),\lambda_a)[1-\phi_F(sPl(\rho))]\rho d\rho d\theta} e^{-sN_0} \tag{1}$$

- ϕ_F - Laplace Transform of the fading random variable; $\phi_F(x) = \frac{1}{1+x}$ as fading is exponentially distributed

Evaluation of equation (1) at $s = \frac{\mu\beta}{\mathrm{P_{AP}}\rho(r)}$ yields $\mathbb{P}[\mathrm{SINR} > \beta]$

Expected Rate Determination

From the CDF of SINR, the expected rate can be computed as:

$$Expected\,Rate = \mathbb{E}[log_2(1 + \mathrm{SINR})]$$