INVESTING IN SHARED SPECTRUM

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Motivation: Tiered Sharing

Interested in case where service providers (SPs) at both tiers compete in same market.
Sharing, Competition & Investment

More competition good for consumers

But may lower firm profits, lower incentives to invest.
Tiered Sharing

Better incentives for higher tier firms

But may further lower them for lower tier
Tiered Sharing

This works seeks to understand these issues.
Setting

Consider single geographic area, single spectrum band.

Set of service providers

Secondaries can access spectrum when ever primary not using it.
Setting

Set of service providers

Invest in Infrastructure

Compete for customers

customers

PA SP

GAA SP 1

GAA SP n
Approach

• Build on model from [Nguyen et al ’11] for competition with unlicensed spectrum.

• In turn based on models for competition in congested markets (e.g. [Acemoglu, Ozdaglar ’07]).
  • Price competition with non-atomic users.
Agenda

• First assume investment is sunk and focus on price competition.

• Then add investment stage.
Model

Providers
SPs set prices $p_i$ to maximize profit.

$$\pi_i = p_i x_i$$

$p_i$ = price for service
$x_i$ = customers served.

Customers
Choose SP $i$ with minimum delivered price

$$p_i + G_i$$

$G_i$ = congestion of SP $i$. 
Idealized Congestion Costs

**Tier 2 SP:**

\[ G_1 = g(x_1). \]

- Increasing, convex in \( x \).

**Tier 3 SPs:**

\[ G_i = g(X) \]

- \( X = \sum x_i \) (total number of customers served)
Inverse Demand

Assume decreasing, concave.

Delivered price at which $x$ customers served

Assume decreasing, concave.
Market Equilibria

Given prices, customers served must be in a Wardrop Equilibrium.
  • Delivered price for all used bands must be the same and less than that on all unused bands.

Service providers prices must in a Nash equilibria.
  • No way any one provider can increase their profit.
Example

Metrics: Consumer and Social Welfare
Basic Lemma

If there is more than one secondary SP, the equilibrium price of secondary service must be zero.
Implication

Tiered sharing always improves consumer and social welfare compared to non-sharing but reduces firm profits.

Tiered sharing also improves social welfare compared to non-tiered sharing.
Investment

SP’s first decide on investment level $I_i$ at cost $c_i$.

$I_i = 0$ means SP stays out of market.

Congestions costs:

$$G_1 = g\left(\frac{x_1}{I_1}\right) \quad G_i = g\left(\frac{x_1}{I_1} + \frac{X-x_1}{I_i}\right)$$

Pay-offs:

$$\pi_i = p_ix_i - c_iI_i$$
Equal costs

Equilibria always exist and are one of the following types:

1. Primary SP and 1 Secondary SP invest;
2. Primary SP and no Secondary SPs invest;
3. No SPs invest.

In case 2, single SP acts as a monopolist.

As cost increases equilibrium transitions from 1->2->3.
Welfare Comparison

Consumer and Social welfare for linear congestions cost $g(y) = y/B$. 

![Graph showing welfare comparison with bandwidth on the x-axis and welfare on the y-axis, with different lines representing different scenarios labeled as SW $\alpha=0.5$, CW $\alpha=0.5$, SW $\alpha=1$, CW $\alpha=1$, SW Sharing, and CW Sharing.](image-url)
Heterogeneous Costs

4 types of equilibria now possible

Only SP1 Invests $(I_1 > 0, I_2 = 0)$

Sharing $(I_1 > 0, I_2 > 0)$

No One Invests $(I_1 = 0, I_2 = 0)$

Only SP2 Invests $(I_1 = 0, I_2 > 0)$

example for linear congestion costs
Welfare

consumer and social welfare for linear congestions cost $g(y) = y/B$. 

![Graph showing the relationship between Welfare and Bandwidth with different lines representing different scenarios of S1 and S2.]

- Low cost primary
- Low cost secondary
Conclusions

• Considered a few simple models to illustrate interaction of competition, investment and sharing.
• Tiered sharing improves welfare if investment is sunk.
• But with investment, benefits depend on making sure “right” firms are assigned to the “right” tiers.
• Many other issues: different customer types, incentives to innovate, investments to improve technology, ...